A Unified Model for the Compressive Strength of FRP-confined Square and Circular Concrete Columns

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ABSTRACT: It has been gradually recognized in recent years that the corner radius plays a key role in the effectiveness of the confinement that is provided to concrete columns by square-shaped FRP jackets. When the corner radius ratio ρ is defined as $2r/b$, where r is the corner radius and b is the breadth of the cross-section, circular columns are just a special case when ρ = 1. Sharp cornered square columns represent another special case when ρ = 0. It is quite natural and logical that a confined concrete model for columns with a corner radius should degenerate into a model for circular columns when ρ = 1, and into another for sharp cornered square columns when ρ = 0. However, this is not the case in any of the existing models, except for that by Mirmiran et al. (1998), which has been overlooked in recent years. Extensive experimental testing on FRP-confined concrete columns that have a continuous variation of ρ from 0 to 1 has been undertaken by the authors. Based on these experimental findings, a rational procedure is proposed in this work for developing a unified strength model for FRP-confined concrete columns that have an arbitrary corner radius. The model can be degenerated into the two special cases of circular columns and sharp cornered square columns when ρ = 1 and 0, respectively. A database has been established by collecting all of the available experimental results from the literature for both circular and square columns. Comparisons between the test results and the model predictions have been made to demonstrate the accuracy of the unified model.

1 INTRODUCTION

The modeling of the compressive strength and stress-strain behavior of FRP-confined reinforced concrete columns is an important, fundamental, and difficult task for FRP applications in the structural rehabilitation industry. Since the 1980s, there have been continuous efforts in this research area to obtain a more reliable and accurate model. Most of these investigations have concentrated on circular columns, and numerous models have been developed. However, relatively few studies have addressed square columns due to the non-uniform distribution of the confinement stress across the cross-section and the subsequent difficulty involved in the modeling.

Experimental tests have established that the corner radius significantly affects the confinement effectiveness, due to the concentration of confinement stress around the corners. Mirmiran et al. (1998) suggested that FRP in square sections is less effective in confining concrete than its circular counterparts. Rochette and Labossière (2000) stated that the confinement effect is directly related to the shape of the section for a given number of wraps around the column. The ratio of the corner radius to the side length of a square section, which takes values between 0 and 0.5, can be used to determine the degree of confinement that can potentially be provided to the concrete by the composite wraps. A test that was conducted by Yang et al. (2004) indicated that a smaller corner radius can significantly reduce the ultimate strength of the FRP laminate due to the stress concentration around the corner area. This stress concentration factor increases when the corner radius decreases. Al-Salloum (2006) investigated the compressive strength of FRP-confined concrete square columns with a series of corner radii that provided a cross-section from square to circular and observed that the best performance was that of the circular specimens. As the shape changed away from circular, that is, from the large corner radius to the small one, the load capacity decreased.

According to the latest research by Wu et al. (2006), the corner radius ratio, ρ, which is defined as the ratio of the corner radius to the half breadth of the column, $2r/b$, is the single most important factor that affects the confinement effectiveness across the cross-section. The authors have undertaken comprehensive experimental tests on FRP-confined concrete square columns that have a continuous variation of ρ from the sharp corners to the circular sections to study the corner radius effect (Wu et al., 2006; Wang & Wu, 2008). The test results show that the strength gain of the columns that is due to FRP confinement has a fairly linear relationship with the corner radius ratio ρ and that there is no significant strength gain for columns with sharp corners. Based on these experimental findings, this work proposes a rational procedure for developing a unified strength model for FRP-confined concrete columns that have an arbitrary corner radius.

2 EXISTING MODELS

2.1 *Models for FRP-confined circular columns*

In a circular concrete column, the confining pressure is constant in the whole cross-section. When the FRP jacket ruptures, this confining pressure reaches its maximum value, which is given by

$$f_l = 2E_{frp}\varepsilon_j t / d \,, \qquad (1)$$

where E_{frp} is the elastic modulus of FRP in the hoop direction, t is the total thickness of FRP, d is the diameter of the confined concrete core, and ε_j is the hoop tensile strain of FRP at failure. At FPR rupture, the hoop strain in the jacket, ε_j, is generally considerably smaller than the ultimate tensile strain obtained from flat coupon tests, ε_{frp}. In the development of confinement models, it is suggested that ε_j be taken as the actual hoop rupture strain, $\varepsilon_{h,rup}$, which can be related to the FRP ultimate tensile strain, ε_{frp}, through an efficiency factor, k_ε, as given by $\varepsilon_{h,rup} = k_\varepsilon \varepsilon_{frp}$. The value of k_ε has been shown to vary with the type of FRP, and an average value of 0.586 has been recommended for carbon FRP-confined circular columns, based on an analysis of the test database by Lam and Teng (2003).

Most of the available models for evaluating the compressive strength of FRP-confined concrete circular columns are based on the confinement model that was derived experimentally by Richart et al. (1928) for specimens under active hydrostatic pressure:

$$f'_{cc} / f'_{co} = 1 + k_1 \left(f_l / f'_{co} \right), \qquad (2)$$

where f'_{cc} and f'_{co} are the compressive strength of confined and unconfined concrete, respectively, f_l is the lateral hydrostatic pressure, and k_1 is the confinement effectiveness coefficient. This is the general form adopted by the majority of the existing strength models for FRP-confined concrete. Numerous strength models have been developed by fitting experimental data to the general form of Equation 2, and the confinement effectiveness coefficient k_1 is derived as a constant, as a function of the effective lateral confining pressure f_l, or as a function of f_l / f'_{co}.

2.2 *Models for FRP-confined square columns*

Four existing typical models for square/rectangular columns with rounded corners are summarized below.

Mirmiran et al. model (1998)

The model proposed by Mirmiran et al. (1998) is given by

$$f'_{cc} / f'_{co} = 1 + k_1 k_s \left(f_l / f'_{co} \right), \qquad (3)$$

where k_s is the shape factor that accounts for the effect of non-uniform confinement and is defined as $k_s = 2r/D$, in which r denotes the corner radius, and D is the diameter of an equivalent circular column and equated to the side length of a square column or the longer side length in the case of a rectangular section. The confinement effectiveness coefficient k_1 in the model is adopted as $k_1 = 6.0 f_l^{-0.3}$. Therefore, its final form is given by

$$f_{cc}^{'} / f_{co}^{'} = 1 + 6.0\left(2r/D\right) f_l^{0.7} / f_{co}^{'} . \tag{4}$$

Lam and Teng's model (2003)

This model takes the form

$$f_{cc}^{'} / f_{co}^{'} = 1 + 3.3\left(A_e / A_c\right)\left(f_l / f_{co}^{'}\right), \tag{5}$$

where the value of the confinement effectiveness coefficient k_1 of 3.3 is obtained by calculating the confining pressure f_l in Equation 1 with $\varepsilon_j = \varepsilon_{h,rup}$ and replacing d with an equivalent diameter D that is defined as the diagonal distance of the section, i.e., $D = \sqrt{h^2 + b^2}$, where h and b are the depth and breadth of the column section, respectively. The shape factor is taken into account by the effective confinement area ratio of A_e/A_c that is given by

$$A_e / A_c = 1 - \left[\left(b/h\right)\left(h - 2R_c\right)^2 + \left(h/b\right)\left(b - 2R_c\right)^2\right] / 3A_g , \tag{6}$$

where A_g is the gross area of the column section, i.e., $A_g = bh - \left(4 - \pi\right) r^2$.

Ilki et al. model (2004)

Ilki et al. (2004) proposed the following model.

$$f_{cc}^{'} / f_{co}^{'} = 1 + 2.4\left(f_{l\max}^{'} / f_{co}^{'}\right)^{1.2} , \tag{7}$$

where $f_{l\max}^{'}$ is the maximum effective lateral confinement stress and can be obtained by

$$f_{l\max}^{'} = \left(\kappa_a \rho_f \varepsilon_{h,rup} E_{frp}\right) / 2 , \tag{8}$$

where ρ_f is the FRP ratio of the cross-section, and $\varepsilon_{h,rup}$ is assumed to be 70% of the ultimate tensile strain of FRP determined from the flat coupon test, i.e., $\varepsilon_{h,rup} = 0.7\varepsilon_{frp}$. The effectiveness factor κ_a is assumed to be 1.0 for circular columns. For rectangular sections, it is given by

$$\kappa_a = 1 - \left[\left(b - 2r\right)^2 + \left(h - 2r\right)^2\right] / 3bh - \left[\left(4 - \pi\right) r^2 / bh\right] - \rho_g , \tag{9}$$

where ρ_g is the ratio of the area of the longitudinal steel reinforcement to the cross-sectional area of the member.

Al-Salloum's model (2006)

In the model proposed by Al-Salloum (2006), the shape factor k_s is expressed as a function of the breadth of the section, b, and the corner radius r as

$$k_s = 1 - 2\left(1 - 2r/b\right)^2 / 3\left[1 - (4 - \pi)\left(r/b\right)^2\right] . \tag{10}$$

As the shape factor k_s takes account of the effective confined area of the section, it is suggested that another factor of b/D that accounts for the non-uniformity of the confining pressure be included, where D is the diagonal length of the square columns, as given by $D = \sqrt{2}b - 2r\left(\sqrt{2} - 1\right)$. The confinement effectiveness coefficient k_1 is taken as 3.14, which is the average value of 2.98 and 3.3, as proposed by Miyauchi et al. (1999) and Lam and Teng (2003), respectively. Therefore, the model is expressed as

$$f_{cc}^{'} / f_{co}^{'} = 1 + 3.14 k_s \left(b/D\right)\left(f_l / f_{co}^{'}\right) . \tag{11}$$

3 EVALUATION OF EXISTING MODELS

The criterion used to evaluate the performance of a model is the mean of the error squares, ω, which is defined as

$$\omega = \sum_1^n \left[Expe.(f_{cc}^{'} / f_{co}^{'}) - Theo.(f_{cc}^{'} / f_{co}^{'})\right]^2 / n , \tag{12}$$

where $Expe.\left(f'_{cc}/f'_{co}\right)$ and $Theo.\left(f'_{cc}/f'_{co}\right)$ are the experimental and theoretical values of strength gain, respectively, and n is the total number of tests.

A database containing the test results of 285 FRP-wrapped plain concrete circular columns and 170 square columns was assembled from the open literature. Of these, 18 circular and 72 square specimens were from the experimental tests carried out by the authors. The circular section specimens included in the database have diameters (d) of from 70 to 200 mm. For the square sections, the section widths (b) are from 94 to 458 mm, and the corner radii (r) are from 0 to 60 mm (i.e., ρ varies from 0 to 0.8). The unconfined concrete strengths range from 18 to 169.7 MPa and 6.8 to 56.7 MPa for the circular and square columns, respectively. Different types of FRP were used in the database specimens, namely CFRP, GFRP, and AFRP. This database and the criterion ω are used together to assess the performance of a model.

For circular models, the model proposed by Karabinis and Rousakis (2001) in the form of

$$f'_{cc}/f'_{co} = 1 + 2.1\left(f_l/f'_{co}\right)^{0.87} \tag{13}$$

is found to fit the database best, with the lowest value of $\omega = 0.081$. The performance of this model is shown in Figure 1.

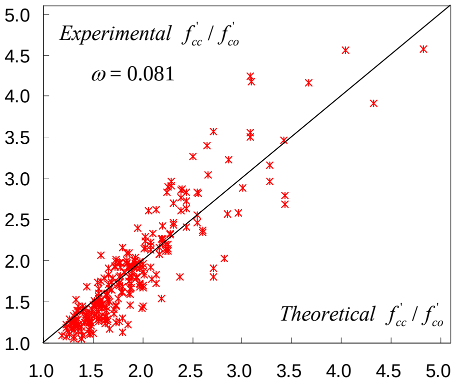


Figure 1 Performance of Equation 13

Figure 2 shows the performances of the four models for square columns that are listed in Section 2.2. The middle line of 1.0 indicates that the predicted values of f'_{cc}/f'_{co} are equal to those of the tests. Data from different sources (papers) are illustrated with different symbols in Figure 2. The error $\omega 1$ in the figure gives the error of the square columns, and $\omega 2$ is the error of all of the columns, including the circular columns.

In the literature, it is common practice for two different models to be used for the circular columns and the square columns. However, it is quite natural and logical that a confined concrete model for columns with an arbitrary corner radius should degenerate into a model for circular columns when ρ = 1, and similarly, into another for sharp cornered square columns when ρ = 0. This is the case for none of the existing models, except for that of Mirmiran et al. (1998). Figure 2 shows that the prediction of the Mirmiran et al. model (1998) presents a certain conservatism at ρ = 0 for sharp cornered square columns and a large overestimation at ρ = 1 for circular specimens. All of the other models overestimate the results for ρ = 0, and if they are extrapolated to circular columns at ρ = 1, then their performance is very poor. Therefore, a unified model for the compressive strength of FRP-confined concrete that covers the full range of ρ = 0 to 1, and hence is applicable to both square and circular columns, is not yet available.

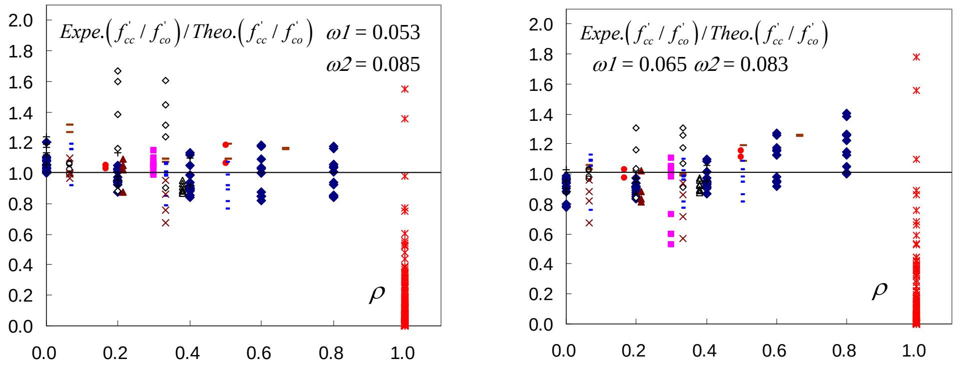
4 PROPOSED MODEL

This work aims to develop a new strength model that is derived from and applicable to the full range of ρ = 0 to 1. Mathematically speaking, a model derived from such a full range of parameters should be more accurate. There are many parameters that affect the strength gain, f'_{cc}/f'_{co}, for square columns. However, when all other factors are fixed, and the only variable considered is ρ, the mathematical form of the strength gain can be expressed as

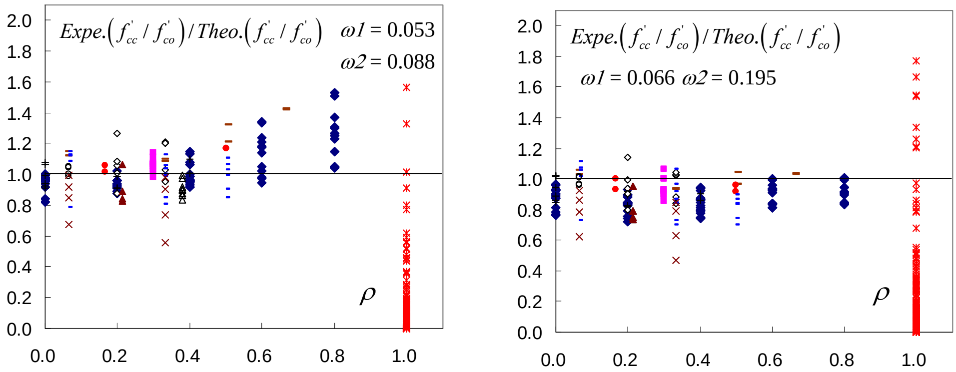
$$f'_{cc}/f'_{co} = 1 + C \cdot f(\rho), \tag{14}$$

where $f(\rho)$ is the shape factor that reflects the effect of the corner radius.

Experimental tests by the authors have found that the strength gain is approximately proportional to the corner radius (Wang & Wu, 2008). Analytical study has also proved that a zero corner radius does not provide any confinement (Wu et al., 2006). Based on these two conclusions, the shape factor can be assumed to have the following form.



(a) Mirmiran et al. model (1998) (b) Lam & Teng’s model (2003)



(c) Ilki et al. model (2004) (d) Al-Salloum’s model (2006)

Figure 2 Performance of existing models for square and circular columns

$$k_s = f(\rho) = \rho^{\alpha}, \tag{15}$$

where α is a constant to be determined, and k_s = 0 and 1 for sharp cornered and circular columns, respectively. Therefore, $f'_{cc}/f'_{co} = 1 + C$ is the strength model for circular columns.

A model similar to Equation 13 is adopted for circular columns in this work. However, the coefficients are further adjusted by regression analysis using the assembled database. The following result is obtained.

$$f'_{cc}/f'_{co} = 1 + 2.16\left(f_l/f'_{co}\right)^{0.955}. \tag{16}$$

The performance of Equation 16, which is shown in Figure 3, is even better than that of Equation 13 which is shown in Figure 1. Substituting Equations 15 and 16 into Equation 14, the strength model for square columns is obtained:

$$f'_{cc}/f'_{co} = 1 + 2.16\rho^{\alpha}\left(f_l/f'_{co}\right)^{0.955}. \tag{17}$$

By regression analysis using the database, α is calculated to be 0.651. Equation 17 becomes

$$f'_{cc}/f'_{co} = 1 + 2.16\rho^{0.651}\left(f_l/f'_{co}\right)^{0.955}. \tag{18}$$

In the above equation, the confining pressure, f_l, is calculated by

$$f_l = 2 f_{frp}\, t/b = 2E_{frp}\varepsilon_{frp}\, t/b, \tag{19}$$

where f_{frp} is the FRP tensile strength in the hoop direction obtained from the flat coupon test.

The performance of the proposed model is shown in Figure 4. The errors of $\omega 1$ = 0.039 and $\omega 2$ = 0.064 are much smaller than the lowest values of the existing models. It can be observed that most of data are evenly distributed around the middle line, including the circular columns at ρ = 1. Although there is a certain underestimation of sharp cornered square columns at ρ = 0, this error may be caused by experimental inaccuracy, because it is impossible in practice to make a zero corner radius, and a small corner radius is often considered to be a sharp corner in many tests.

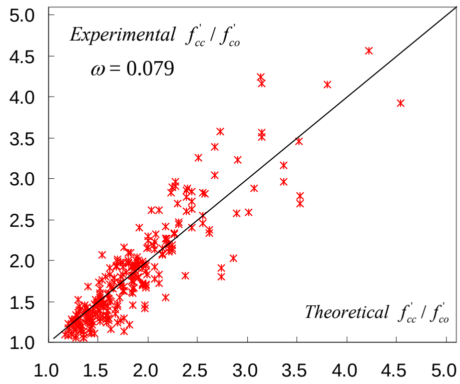


Figure 3 Performance of Equation 16

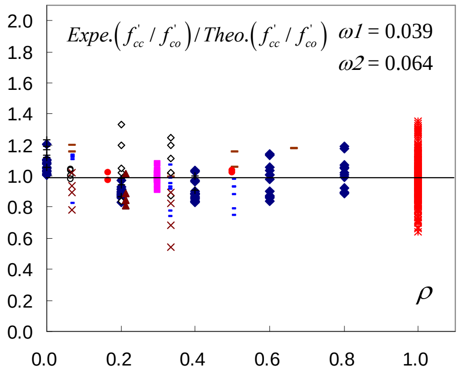


Figure 4 Performance of Equation 18

5 CONCLUSIONS

A unified model for the strength of FRP-confined concrete columns that have an arbitrary corner radius is developed by introducing a shape factor that is expressed as a function of the corner radius ratio ρ. The model can be degenerated into two special cases for circular columns and sharp cornered square columns when $\rho = 1$ and 0, respectively. A database has been established by collecting all of the available experimental results on both circular and square columns from the literature for model evaluation. Comparisons between the test results and the model predictions have demonstrated the accuracy and superior performance of the proposed model.

6 ACKNOWLEDGEMENTS

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