Analytical modeling of square and rectangular concrete sections confined by FRP: ultimate strength prediction

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ABSTRACT: An analytical model tailored for the ultimate stress prediction of the FRP confined concrete section is presented. The model has been conceived to be implemented in a design code and a simple yet robust mechanics-based equation has been developed and tested against a set of experimental literature data. The model is thought for square/rectangular sections and considers the corner radius influence, so that circular sections are automatically included as a particular case.

1 INTRODUCTION

Reliable prediction of ultimate condition for FRP confined section is a fundamental step when implementing design equations. For both ultimate strain and stress a lot has been done for circular sections, while few advances have been reported for square and rectangular ones: for the ultimate strength prediction the authors proposed an analytical model tailored for square section. Since it depends on the corner radius, circular sections are included as a particular case. The basic assumptions of the model are: 1) the ultimate stress is independent of the confinement path, and 2) the confinement internal stress field is independent of concrete Young modulus, under a given boundary condition given in terms of confinement force. The model, as well as its assumptions, have been already discussed in (Monti and Nisticò, 2007); in the following sections, after a preliminary explanation of the approach adopted for square sections, its extension to rectangular sections will be presented.

2 ULTIMATE STRENGTH PREDICTION

In order to evaluate the ultimate strength, it is expedient to express the Confinement Stress Field (CSF) in terms of normalized quantities, *i.e.*, the ratio (PSR = Principal Stress Ratio) between the minimum (σ_{min}) and maximum (σ_{max}) principal stresses, and the ratio (NMPS = Normalized Maximum Principal Stress) between σ_{max} and the constant confinement stress (f_l) evaluated in a circular section with radius $R = 0.5 \cdot L_{min}$ (compression stresses are considered as positive). Parametric numerical investigations have been carried out in order to capture the trend of both PSR and NMPS: their spatial distribution depends on the section aspect ratio (here defined as the ratio between the long, L_{max}, and short, L_{min}, section side: $S = L_{max}/L_{min}$) as reported in the following sections, where three fundamental cases will be analysed: 1) square section, 2) rectangular section with $S \geq 2$, and 3) rectangular section with $S \leq 2$.

2.1.1 *Square section*

In order to define the CSF, two sub-domains can be identified (see Figure 1a): 1) the *core*, that corresponding to the area of the inscribed circle, 2) the *corner*, corresponding to the area between the *core* and the section perimeter. It can be observed (Figure 1b), as already reported in (Monti and Nisticò, 2007), that:

- inside the *core* the NMPS is practically constant and equal to 1, b) the PSR is 1 at the centre and tends to 0 close to the *core* boundaries;
- inside the *corner* a) the NMPS reaches its *max* absolute value at the section edge and its *min* absolute value at the core boundaries; b) the PSR can attain (depending on the *corner* r*ounding*) negative values; c) at the corner centroid the PSR can assume a value greater (and not lower) than the *core* PSR;
- clearly, the CSF depends on the ratio between the *corners* rounding radius r_c and the *core* radius R (half of the section side: $R = 0.5\ L$): both NMPS and PSR tend to 1 if the *corner* radius tends to the *core* radius.

Based on the previous assumptions: a) the *core* maximum principal stress, $(\sigma_{max})_{core}$, is assumed equal to the stress field in a circular section having the same wrapping in terms of width and mechanical properties; b) the *corner* maximum principal stress, $(\sigma_{max})_{corner}$, depends (if evaluated at the *corner* centroid) on the corner radius (r_c) as follows:

$$(\sigma_{max})_{corner} = \left(2 - 2\cdot\frac{r_c}{L}\right)^{\beta}\cdot(\sigma_{max})_{core} \tag{1}$$

The previous expression states that: 1) if r_c tends to $0.5\cdot L$, $(\sigma_{max})_{corner}$ tends to $(\sigma_{max})_{core}$ since the square section becomes circular, 2) if $r_c < 0.5\cdot L$, $(\sigma_{max})_{corner}$ depends on the parameter ß, which can be obtained from statistical elaboration of experimental data.

At the *corner* centroid, the PSR is assumed to depend on the corner/core radii ratio, as:

$$\alpha_{corner} = 1 - \gamma\cdot\left(1 - 2\cdot\frac{r_c}{L}\right) \tag{2}$$

The previous expression implies that: 1) if r_c tends to $0.5\cdot L$, α_{corner} tends to 1 (equal confinement stresses); 2) if $r_c = 0$, α_{corner} is equal to $(1-\gamma)$, where the γ parameter can be obtained from statistical elaboration of experimental data.

The evaluation of the *core* PSR can be assumed as inversely proportional to the distance d of the considered point from the *core* centre (having an axially symmetric field), according to the following expression:

$$\alpha_{core} = \frac{\sigma_{min}}{\sigma_{max}} = 1 - 2\frac{d}{L}\cdot\left(1 - 2\cdot\frac{r_c}{L}\right) \tag{3}$$

The previous expression implies (Figure 2) that α_{core} is equal to 1 either at the centre of the *core* or everywhere if $r_c = 0.5\cdot L$ (that is the case of the circular section).

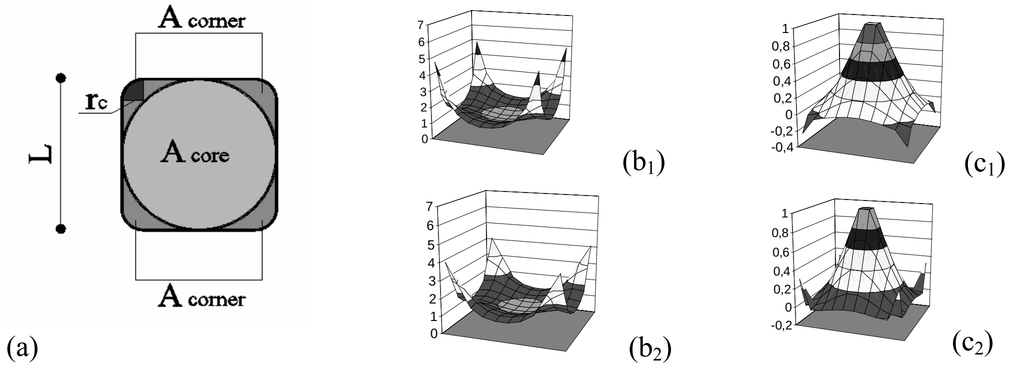
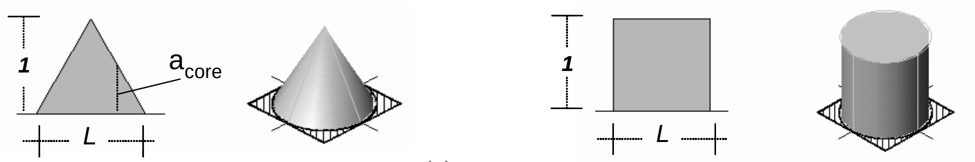


Figure 1. Square section: region identification (a); numerically evaluated NMPS (b) and numerically evaluated PSR (c). The reported stress field relates to the cases of $r_c = 0$ (b_1, c_1) and $r_c = 0.2L$ (b_2, c_2).

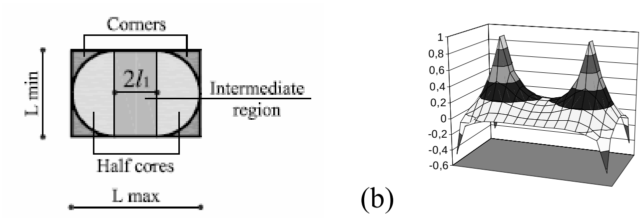


(a) (b)

Figure 2. Square section. Assumed *core* PSR: $r_c = 0$ (a) and $r_c = 0.5L$ (b).

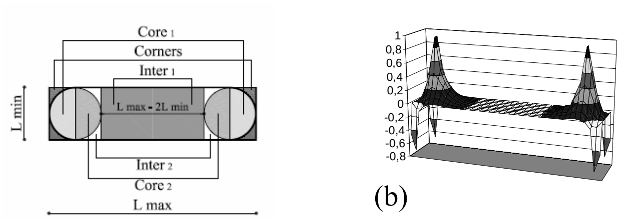
2.1.2 *Rectangular section*

As already specified, the PSF distribution depends on the above-defined section aspect ratio *S*. In order to define homogeneous regions in terms of PSF, different assumptions will be made by distinguishing different sub-domains for the two analyzed cases ($S \leq 2$ and $S \geq 2$).
If $S \leq 2$, three sub-domains (Figure 3a) can be identified: 1) two half-cores, 2) the corners, and 3) an intermediate region. The diameter of the two half-cores is equal to L_{min}: each centre is distant $0.5 \cdot L_{min}$ from the nearest shortest section side. The *corner* sub-domain corresponds to the area bounded by the two half-*cores* boundaries and the section perimeter. The intermediate sub-domain, in between the two half-*cores*, corresponds to a rectangular region whose length ($2 \cdot l_1$) ranges between zero, when $S = 1$, and L_{min}, when $S = 2$.
If $S \geq 2$, together with the *corner* and the two half-*core* regions (here called *core*$_1$), three other sub-domains will be identified; they are: a) an intermediate region (here called *inter*$_1$), whose length is ($L_{max} - 2 \cdot L_{min}$), the two other half-cores (here called *core*$_2$) and the remaining region (here called *inter*$_2$) that lies between the *core* region and the intermediate one.



(a) (b)

Figure 3. Rectangular section: $S \leq 2$; sub-domains (a) and PSR (b).



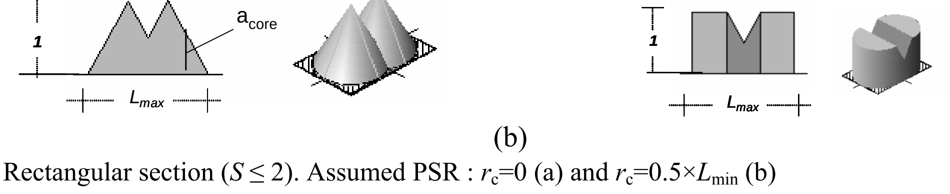
(a) (b)

Figure 4. Rectangular section. $S \geq 2$: sub-domains (a) and PSR (b).

Independently of the aspect ratio, 1) the *corner* CSF can be assumed as that already described for the square region, and 2) outside the *corner* region the NMPS is practically constant and equal to 1. Regarding the outside corner region the following PSR can be assumed depending on the aspect ratio.
If $S \leq 2$ the two half-*cores* are characterized by PSR almost equal to that assumed for the square *core*, while the intermediate region is in general characterized by a non-simple distribution, consequent to the interaction between the two cores (Figure 3b and 5a). If $r_c = 0.5 \cdot L_{min}$, it is possible to assume that the distribution has the shape reported in Figure 5b, where it is possible to note that the two half-cores are characterized by a unitary constant value, while the other two confined parts are characterized by a decreasing value according the following expression:

$$\alpha_{\underline{inter}} = \left(1 - 2\frac{l}{L_{min}}\right) \tag{4}$$

where l (whose value ranges between 0 and l_1) is the distance of the considered point from the segment passing from the *core* centre and perpendicular to the two longer sides.

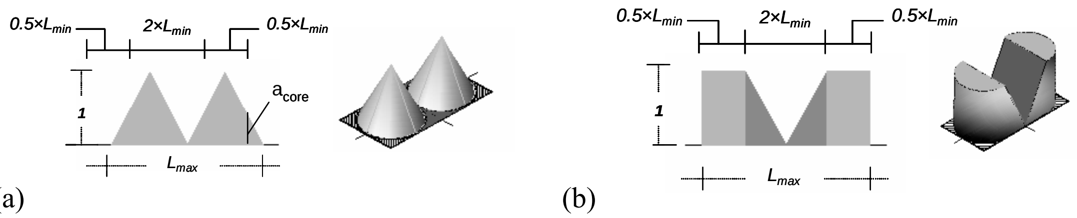


(a) (b)

Figure 5. Rectangular section ($S \leq 2$). Assumed PSR : r_c=0 (a) and r_c=0.5×L_{min} (b)

If S ≥ 2, 1) the two half $core_1$ regions are characterized by a PSR almost equal to that assumed for the square *core*, 2) one of the two intermediate ($inter_1$) region is characterized by mono-dimensional CSF so that $\sigma_{min} = 0$. For the other regions, the CSF can be assumed as follows, distinguishing two extreme situations that are $r_c = 0$ and $r_c = 0.5 \cdot L_{min}$:

- if $r_c = 0$, the $core_2$ PSR can be assumed equal to that assumed for the $core_1$, and $inter_2$ region is characterized by mono-dimensional CSF so that $\sigma_{min} = 0$;
- if $r_c = 0.5 \cdot L_{min}$, $core_2$ and $inter_2$ regions can be grouped in one region characterized by a decreasing value of the PSR as expressed through Equation 4.



(a) (b)

Figure 5. Rectangular section (S = 2): assumed min/max PSR.

2.2 *Evaluation of the section ultimate strength*

Under the assumption of path-independence, if the CSF is known at the ultimate condition (*e.g.*, FRP failure), then the corresponding ultimate strength can be evaluated in each region of the section, based on a selected strength criterion here assumed as follows:

$$f_{cc} = C_1 \cdot f_{co} + F(\sigma_{max}) \cdot \frac{\sigma_{min}}{\sigma_{max}} \tag{5}$$

where $F(\sigma_{max})$ is a function giving the strength increase when $\sigma_{min} = \sigma_{max}$ (*i.e.*, in circular sections).

By integrating the previous expression over the section area, the global ultimate strength can be found by evaluating the ultimate resisting force as follows:

$$F_{cc} = F_{co} + \int_{A_{section}} F(\sigma_{max}) \cdot \frac{\sigma_{min}(\bullet)}{\sigma_{max}(\bullet)} \cdot dA = F_{co} + \int_{A_{section}} F(\sigma_{max}) \cdot \alpha(\bullet) \cdot dA \tag{6}$$

where $F_{co} = C_1 \cdot f_{co} \cdot A_{sec}$ is the unconfined section ultimate resisting force. In order to obtain a viable expression for it, the simplifications, already discussed (see § 2.1), are introduced in the definition of the CSF, depending on the aspect ratio S.

In case of *square section*, the strength increase has to be differentiated for either of the two pertinent regions that are *core* and *corner*; so that:

$$f_{cc} = \frac{(F_{co})_{square} + (\Delta F_{cc})_{core} + (\Delta F_{cc})_{corner}}{(A_{sec})_{square}} \tag{7a}$$

$$(\Delta F_{cc})_{corner} = \int_{A_{corner}} \left[F(\sigma_{max})_{corner} \cdot \alpha_{corner} \right] \cdot dA = A_{corner} \cdot \left[F(\sigma_{max})_{core} \cdot \left(1 - \gamma + 2\gamma \frac{r_c}{L} \right) \right] \tag{7b}$$

$$(\Delta F_{cc})_{core} = \int_{A_{corner}} \left[F(\sigma_{max})_{core} \cdot \alpha_{core} \right] \cdot dA = A_{core} \cdot \left[\frac{1}{3} F \cdot (\sigma_{max})_{core} \cdot \left(1 + 4 \cdot \frac{r_c}{L} \right) \right] \tag{7c}$$

In case of *rectangular section* with S ≥ 2, in order to evaluate the ultimate resisting force by means of an expression formally consistent with the assumed distribution, the following expression has been developed:

$$\Delta F_{cc} = (\Delta F_{cc})_{corner} + (\Delta F_{cc})_{core} + G_1(r_c) \tag{8a}$$

where $(\Delta F_{cc})_{corner}$ and $(\Delta F_{cc})_{core}$ can be evaluated through expressions 7 (b,c), and $G_1(r_c)$ through the following expression:

$$G_1(r_c)=\left[L^2{}_{min}\cdot\frac{\pi}{4}\cdot\frac{1}{3}\cdot F\left(\sigma_{max}\right)_{core}\right]+F\left(\sigma_{max}\right)_{core}\cdot L^2{}_{min}\cdot\left[\frac{1}{2}-\frac{\pi}{4}\cdot\frac{1}{3}\right]\cdot 2\cdot\frac{r_c}{L_{min}} \tag{8b}$$

that can be approximated as follows, assuming Π ~ 3:

$$G_1(r_c)=\left[L^2{}_{min}\cdot F\left(\sigma_{max}\right)_{core}\right]\cdot\frac{1}{4}\cdot\left(1+2\cdot\frac{r_c}{L_{min}}\right) \tag{8c}$$

The previous expression states (having introduced the $G_1(r_c)$ function) that if $r_c = 0$, the strength increase can be obtained by considering the contribution of two cores, while approaching r_c the value of $0.5\cdot L_{min}$ the contribution of one core has to be replaced with the contribution consistent with the pertinent min/max PSR distribution (see Figure 5b and Equation 4).
In case of *rectangular section* with S ≤ 2, in addition to the *corner* region and the two half *core* regions, the contribution of the intermediate zone has to be considered. If $r_c = 0.5\cdot L_{min}$, this contribution can be evaluated as follows:

$$\left(\Delta F_{cc}\right)_{\underline{int}}=2\cdot L_{min}\cdot F\left(\sigma_{min}\right)_{core}\int_0^{l_1}\left(1-2\cdot\frac{l}{L_{min}}\right)\cdot dl= \tag{9a}$$

$$=\frac{1}{2}\cdot F\left(\sigma_{min}\right)_{core}\cdot\left(3\cdot L_{min}-L_{max}\right)\cdot\left(L_{max}-L_{min}\right)$$

In order to generalize Equation 9a, introducing the dependence on the corner radius, a new function is defined so that:

$$\left(\Delta F_{cc}\right)_{int}=\left(\Delta F_{cc}\right)_{\underline{int}}+G_2\left(r_c\right) \tag{9b}$$

where the new introduced function has to fulfil the following conditions: 1) it has to be zero if $S = 1$ and it has to be such that the ultimate resisting section strength evaluated by means of Equation 9b be equal to the value obtainable from Equation 8a, if $S = 2$. So that, the sought function can be evaluated according to the following expression:

$$G_2\left(r_c\right)=\left[\left(\frac{1}{2}-\frac{1}{3}\cdot\frac{\pi}{4}\right)\cdot\left(L_{min}-L_{max}\right)\cdot L_{min}\cdot F(\sigma_{max})_{core}\right]\cdot\left[1-2\cdot\frac{r_c}{L_{min}}\right] \tag{9c}$$

that can be approximated as follows, assuming Π ~ 3:

$$G_2\left(r_c\right)=\left[\frac{1}{4}\cdot\left(L_{min}-L_{max}\right)\cdot L_{min}\cdot F(\sigma_{min})_{core}\right]\cdot\left[1-2\cdot\frac{r_c}{L_{min}}\right] \tag{9d}$$

3 RESULTS AND DISCUSSIONS

Equations 7,8,9 define the ultimate strength for square and rectangular section as function of: 1) the parameters β and γ (introduced for the definition of the corner stress distribution), 2) the parameter C_1 introduced for the definition of the unconfined strength, and 3) $F(\sigma_{min})$ that can be assumed as proportional to the minimum principal confinement stress (f_{lu}) as:

$$F\left(\sigma_{\max}\right)=C_2\cdot f_{lu}\cdot\left(\frac{r_c}{R}\right)^{C_3} \tag{10}$$

where $(r_c/R)^{C_3}$ represents a reduction factor accounting for the ultimate strain reduction when considering coupon test failure strains (for sharp corner radius, $r_c = 0$, the confinement is assumed as inefficient). A calibration of the parameters (β = 1.7, γ = 1, $C_1 = 0.8$, $C_2 = 3$; $C_3 = 0.5$) has been performed in (Monti and Nisticò, 2007) where, further on, it has been deduced that for the *corners* the *core* ultimate stress expression can be assumed.
Having defined the parameters, based on the set of proposed expressions (7,8,9) the confined strength can be expressed as follows for the three considered cases ($S = 1$, $S \geq 2$, $S \leq 2$):

$$f_{cc} = 0.8 \cdot f_{co} + f_{lu}^* \cdot \left(1 + 4 \cdot \frac{r_c}{L}\right) \qquad if\ S = 1 \qquad (11a)$$

$$f_{cc} = 0.8 \cdot f_{co} + f_{lu}^* \cdot \left[\frac{A_{square}}{A_{section}} \cdot \left(1 + 4 \cdot \frac{r_c}{L_{\min}}\right) + A_1 \frac{3}{4} \frac{L_{\min}^2}{A_{section}} \cdot \left(1 + 2 \cdot \frac{r_c}{L_{\min}}\right)\right] \qquad if\ S \le 2 \qquad (11b)$$

$$f_{cc} = 0.8 \cdot f_{co} + f_{lu}^* \cdot \left[\frac{A_{square}}{A_{section}} \cdot \left(1 + 4 \cdot \frac{r_c}{L}\right) + \frac{3}{4} \cdot \frac{L_{\min}^2}{A_{section}} \cdot \left(1 + 2 \cdot \frac{r_c}{L_{\min}}\right)\right] \qquad if\ S \ge 2 \qquad (11c)$$

where:

$$A_1 = 2 \cdot \frac{(L_{\max} - L_{\min})}{(L_{\min} + 2 \cdot r_c)} \cdot \left[\frac{5}{2} + \frac{r_c}{L_{\min}} - S\right]; \quad f_{lu}^* = 2 \cdot \frac{t_f}{L_{\min}} \cdot E_f \cdot \varepsilon_l \cdot \left(2 \cdot \frac{r_c}{L_{\min}}\right)^{0.5} \qquad (11d)$$

The proposed expressions fit well the experimental results presented in a companion paper (Monti and Nisticò, 2008). The evaluated errors are reported in Table 1 in terms of Average absolute percentage Error (AE), and Average Ratio (AR) between predicted and experimental values (the errors have been specified distinguishing the set of square, rectangular and circular section here intended as section with $r_c \geq 0.25L$).

Table 1. Predictive equations: evaluated errors

	Circular	Square	Rectangular	All
N_{tests}	28	48	5	81
AE	9.2	33.5	19.50	24.00
AR	1.1	1.35	1.10	1.25

4 CONCLUSIONS

This paper presented an analytical model conceived to predict the ultimate strength of FRP-confined square/rectangular sections with different corner rounding radii. The model is based on: 1) a *path-independence* assumption, 2) a simplified analytical description of the non-uniform confinement stress field, and evaluates the ultimate strength through a simple strength criterion. The proposed model has been satisfactorily validated against the results of 81 selected experimental tests.

5 ACKNOWLEDGEMENTS

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6 REFERENCES

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