Calculation of load-bearing capacity of RC columns strengthened with CFRP materials

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ABSTRACT: The proposal of modification in formula, which allows to calculate load-bearing capacity of axial compressed RC columns strengthened with CFRP strips and sheets, is presented in the paper. Proposed modifications are based on the experimental investigations. Additionally the example of calculation is presented. Authors conducted reliability analysis of described model.

1 INTRODUCTION

Strengthening of RC columns with wrapping in an external composite CFRP jacket causes three-axial state of stress in cross-section and limitation of crosswise strains. Moreover, the experimental investigations (Trapko 2003, 2004; Kamiński & Trapko 2006) proved that use of additional longitudinal strengthening (CFRP strips) restricts the speed of increase of longitudinal strains. The limitation of speed of longitudinal strains is directly connected with increase of load-bearing capacity N_u.

In paper the modification and supplement of theoretical model proposed by Campione and Miraglia (2003) were shown. This model is based on description of the relation between state of stress and strains caused with immediate axial load for columns confined with internal spiral steel confinement.

2 DESCRIPTION OF THE OWN MODEL

In case of elements with rectangular cross-section strengthened with external longitudinal CFRP strips load capacity increases. This increase is caused by longitudinal composite reinforcement to a significant extent (Trapko 2003, 2004, Kamiński & Trapko 2006). However, the crosswise bands (CFRP strips) or continuous jacket (CFRP sheets) are of crucial importance. Their task is to retard debonding of longitudinal CFRP strip. The later that happens the load capacity increase is bigger.

In comparison to RC columns without strengthening, increase of mean limit compressive strains is small. Concrete and longitudinal reinforcement bars transfer the same load as RC column. In conclusion the limit longitudinal strains of investigative elements depends on the presence of longitudinal composite strengthening in small degree.

The investigations proved that the value of the limit longitudinal strains $\varepsilon_{v,lim}$ of RC columns (rectangular cross-section) strengthened with longitudinal CFRP strips and wrapped with external composite CFRP reinforcement does not depend on intensity of longitudinal reinforcement. The intensity of longitudinal reinforcement is calculated as CFRP strips area A_L in relation to the concrete element area A_c. For design purposes it is proposed to assume that load capacity is exceed when ε_v reaches 2,20 ‰.

As result of the performed studies authors propose to modify load capacity (N_u) formula for axial compressed RC columns strengthened with CFRP materials to the form as follows:

$$N_u = f'_c \cdot A_c + f_y \cdot A_s + k_1 \cdot k_e \cdot f_l \cdot A_c + \varepsilon_v \cdot E_L \cdot A_L \quad (1)$$

where:
f'_c – cylindrical compressive strength of concrete in one axial state of stress,
A_c – area of wrapped concrete with FRP jacket (in cross-section),
A_s i f_y – area and yield strength of longitudinal reinforcing steel bars,
k_e – coefficient of effectiveness of FRP confinement,
k_1 – coefficient of strength increasing, it depends on concrete properties and side stress intensity; according to Campione and Miraglia (2003) k_i=2,0,
f_l – crosswise stress on concrete core caused with external FRP jacket,
ε_v – longitudinal limit strain in cross-section of strengthened element (strengthened with longitudinal CFRP strips and wrapped with CFRP material) – 2,20 ‰,
E_L – modulus of elasticity of CFRP strips,
A_L – area of CFRP strips cross-section.

The next modification proposed to take into account are general formulas which allow to calculate crosswise stress in contact between concrete and external confinement for rectangular cross-section shape.

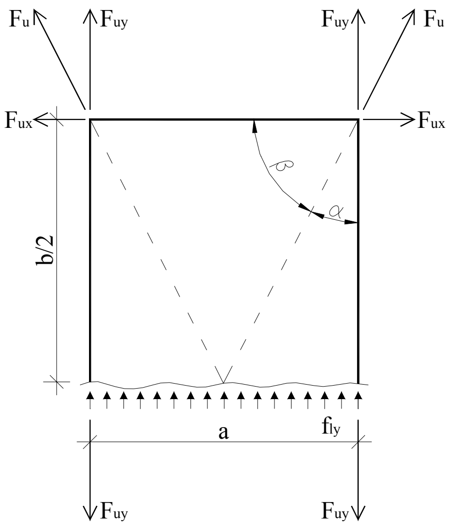

Figure 1. Diagram of forces influence in rectangular cross-section

The perpendicular pressure f_l causes tensile forces F_u in external composite jacket. It arises in contact of concrete and external confinement, the equilibrium equations in cross-section (Figure 1) are as follows:

$$F_{ux} = \frac{1}{2} \cdot f_{ly} \cdot a, \; F_{ux} = F_u \cdot \cos\beta = f_u \cdot t \cdot \cos\beta \quad (2, 3)$$

$$F_{uy} = \frac{1}{2} \cdot f_{lx} \cdot b, \; F_{uy} = F_u \cdot \cos\alpha = f_u \cdot t \cdot \cos\alpha \quad (4, 5)$$

where:
f_{lx}, f_{ly} – crosswise stress in directions x and y,
f_u – tension strength of FRP sheet,
t – nominal thickness of FRP jacket,
a, b – dimensions of cross-section.

After comparison of equations (2) with (3) and (4) with (5) following formulas are achieved:

$$f_{ly} = \frac{2 \cdot f_u \cdot t \cdot \cos\beta}{a} \tag{6}$$

$$f_{lx} = \frac{2 \cdot f_u \cdot t \cdot \cos\alpha}{b} \tag{7}$$

Subsequently, after consideration of cosine of angles α, β and alteration coefficient k_i equations have a form:

$$f_{ly} = \frac{2 \cdot f_u \cdot t}{\sqrt{(a^2 + b^2)}} \cdot k_i \tag{8}$$

$$f_{lx} = \frac{2 \cdot f_u \cdot t}{\sqrt{(a^2 + b^2)}} \cdot k_i \tag{9}$$

where:
k_i – alteration coefficient; according to Campione and Miraglia (2003) k_i=0,2121.

Afterwards authors propose to introduce to the coefficient of effectiveness of FRP confinement k_e formula following modifications:

1) consideration of non-continuity of wraps (locally strengthening with CFRP bands),
2) reduction of concrete core cross-section A_{cc} and effective area of concrete core A_e with area of longitudinal steel reinforcement,
3) consideration of curve radius of cross-section corners in A_{cc} i A_e.

For this purpose equation from Campione and Miraglia (2003) studies is used. Finally, formula, which allows to calculate relation effective area of concrete core A_e to concrete core cross-section A_{cc}, is as follows:

$$k_e = \frac{A_e}{A_{cc}} = \frac{\left(a \cdot b - \sum_{i=1}^{n} \frac{(w_i')^2}{6} - (4-\pi) \cdot r^2 - A_s\right) \cdot \left(1 - \frac{s'}{2 \cdot a}\right) \cdot \left(1 - \frac{s'}{2 \cdot b}\right)}{(a \cdot b - (4-\pi) \cdot r^2 - A_s)} \tag{10}$$

where:
A_e – effective area of concrete core wrapped with FRP jacket,
A_{cc} – area of concrete core reduced with area of longitudinal steel reinforcement,
a, b – dimensions of cross-section,
r – curve radius of cross-section corners,
s’ – distance between CFRP bands,
A_s – area of longitudinal steel reinforcement.

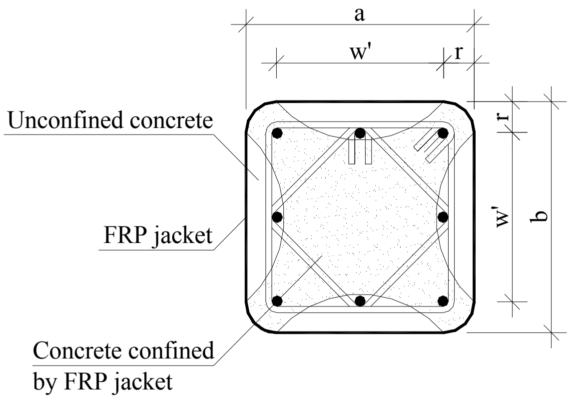

Figure 2. Strengthening effect in column with rectangular cross-section

3 NUMERICAL EXAMPLE

The numerical example is shown below to present the use of the method in practise. Additional part of numerical example is reliability analysis.

3.1 *Deterministic designing*

Geometric data and material properties are listed in the Table 1.

Table. 1. Geometric data and material properties in numerical example

Name of value [Unit]	Characteristic value	Reducing coefficient	Designing value
Concrete strength C30/37 [MPa]	30	1,5	20
Column dimensions (Cross-section) [mm]	200x200	—	200x200
Yield strength of steel A-III (Polish Standard) [MPa]	410	1,15	350
Area of reinforcement 4#12 [mm^2]	452	—	452
Modulus of elasticity of strips Sika Carbodur [GPa]	210	1,2	175
Area of CFRP strips 4 x double M914 [mm^2]	1008	—	1008
Tension strength of sheet Sika Wrap 230C [MPa]	4300	1,2	3583
Thickness of CFRP sheet 3 layers [mm]	0,36	—	0,36

The load capacity of the strengthened column with properties as above, calculated according to formula (1) is 1387 kN (case of axial compression). In example assumed load effects are as follows:

Table 2. Load effects

Type of load	Characteristic value [kN]	Load coefficient [-]	Designing value [kN]
Self weight of structure	545	1,1	600
Serviceable load	625	1,2	750
In total	1170	—	1350

In that conditions the ratio of usage of column is (1350/1387)=97 %.

3.2 *Reliability analysis*

3.2.1 *Statistical properties of variables*

The table 3 presents the statistical properties of variables. In calculation some of them were assumed as deterministic values. Variables in calculations are based on different sources:

1) papers (Nowak 2007; Woliński 2000),
2) Polish Standards (Polish Standard PN-88/B-06250),
3) internet publications, recommendations of CFRP materials producers.

Table 3. Statistical properties of variables

Name of variable [Unit]	Symbol	Mean/Deterministic value(*)	Standard deviation
Concrete strength C30/37 [MPa]	f_c	42	7
Column width (Cross-section) [mm]	a	200,6	5,2
Column height (Cross-section) [mm]	b	200,6	5,2
Yield strength of steel A-III (Polish Standard) [MPa]	f_y	470	60
Area of reinforcement 4#12 [mm^2]	A_s	452,4	3,393
Modulus of elasticity of strips Sika Carbodur [GPa]	E_L	220	6,05
Area of CFRP strips 4 x double M914 [mm^2]	A_L	1008(*)	—
Tension strength of sheet Sika Wrap 230C [MPa]	f_u	5150	515
Thickness of CFRP sheet 3 layers [mm]	t	0,36(*)	—
Self weight of structure [kN]	N_1	545	55
Serviceable load [kN]	N_2	482	86,8

Authors made the simplification because all of random variables have the normal distribution. It seems to be justified for engineering purposes.

3.2.2 *Method of reliability analysis*

Reliability analysis was conducted with Hasofer-Lind method. It is iterative method of calculation of limit state function (11):

$$f(X_1, X_2, \ldots, X_n) = 0 \tag{11}$$

where: $X_1, X_2, \ldots, X_n$ are parameters of load-bearing capacity and load. Limit state function is calculating in design point (12):

$$\mathbf{P} = \{P_1, P_2, \ldots, P_n\} \tag{12}$$

In the first step of iteration as design point are assumed mean values of variables. Authors take mean values for n-1 variables and calculated n^{th} value with formula (11) in example. The next step of procedure is standardisation of variables what gives the vector as follows:

$$\mathbf{U} = \left\{ \frac{P_1 - \mu_1}{\sigma_1}, \frac{P_2 - \mu_2}{\sigma_2}, \ldots, \frac{P_n - \mu_n}{\sigma_n} \right\} \tag{13}$$

where: μ_i, σ_i – mean and standard deviation of i^{th} variable. The vector of gradients in design point is calculated:

$$\mathbf{G} = \{G_1, G_2, \ldots, G_n\} = \left\{ -\left.\frac{\partial F}{\partial X_1}\right|_{\mathbf{X=P}} \cdot \sigma_1, -\left.\frac{\partial F}{\partial X_2}\right|_{\mathbf{X=P}} \cdot \sigma_2, \ldots, -\left.\frac{\partial F}{\partial X_n}\right|_{\mathbf{X=P}} \cdot \sigma_n \right\} \tag{14}$$

That allows to calculate reliability index β for first iteration:

$$\beta = \frac{\mathbf{G}^{\mathbf{T}}\mathbf{U}}{\sqrt{\mathbf{G}^{\mathbf{T}}\mathbf{G}}} \tag{15}$$

To conduct the next iteration it is necessary to calculate vector of coefficients α:

$$\boldsymbol{\alpha} = \frac{\mathbf{G}}{\sqrt{\mathbf{G}^{\mathbf{T}}\mathbf{G}}} \tag{16}$$

The coordinates of design point in the next iteration are:

$$P_i = \mu_i + \alpha_i \, \beta \, \sigma_i \tag{17}$$

The next iteration should be conducted for the new vector **P**. Calculations continue until value of reliability index β stabilizes.

3.2.3 *Presented example*

In the example the limit state function is as follows:

$$f(f_c, a, b, f_y, A_s, f_u, E_L, N_1, N_2) = N_u - (N_1 + N_2) \tag{18}$$

where: N_u – load – bearing capacity of strengthened column (1). Results of calculations are listed in Table 4 and shown on Figure 3.

Table 4. Results of analysis

Number of iterations	1	2	3	4	5
Reliability index β	4,642	4,740	4,737	4,737	4,737

It is seen that reliability index β stabilizes in the third step of iteration. Probability of failure for $\beta = 4,737$ is $P_f = 1,1 \times 10^{-6}$. It is common value for typical engineering structures.

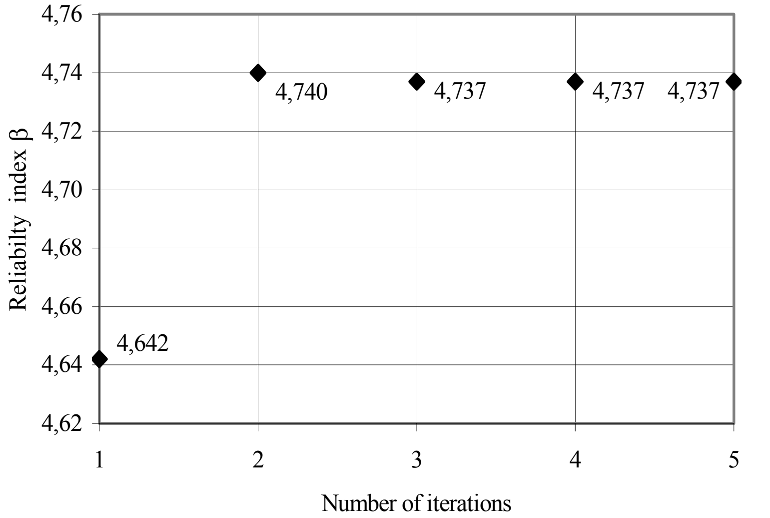

Figure 3. Results of analysis

4 CONCLUSIONS AND FINAL REMARKS

Introduced modifications were verified with the experimental investigations and results of other researchers (e. g. Rochette & Labossière 2000). Verification confirmed agreement of proposed model with results of experimental studies (Trapko 2004, 2006).

Results of reliability analysis let suppose that assumed model is correct. For typically structural elements (beams and columns) reliability index β is between 3 and 4 (Nowak 2007). Slightly higher reliability index β (4,737) received in the calculations can be the consequence of simplifying assumptions (empirical coefficients and dimensions of CFRP reinforcement are deterministic). Furthermore, authors encountered obstacles collecting the information concerning the probability characteristics of CFRP materials. Conducted analysis based on CFRP producers information from Poland and USA. Unfortunately, some of them were not unambiguous.

5 REFERENCES

Campione G., Miraglia N. 2003. Strength and strain capacities of concrete compression members reinforced with FRP. *Cement and Concrete Composites*, No. 25, 31-41.

Joint Committee on Structural Safety, The JCSS Probabilistic Model Code - internet publication (http://www.jcss.ethz.ch/).

Kamiński M., Trapko T. 2006. Experimental behaviour of reinforced concrete column models strengthened by CFRP materials. *Journal of Civil Engineering and Management*, Vol. XII, Issue 2, June, 109-115.

Kamiński M., Trapko T. 2006. On effective use of CFRP as strengthening reinforcement of eccentric compressed RC columns. *Inżynieria i Budownictwa,* No. 1, 39-43 (in Polish).

Nowak A.S. 2007. Risk analysis and reliability assessment for structures. *XXIII Konferencja naukowo-techniczna Awarie Budowlane*, Szczecin-Międzyzdroje, May, 123-130 (in Polish).

Polish Standard PN-88/B-06250. Concrete (in Polish).

Polish Standard PN-B-03264:2002. Plain, reinforced and prestressed concrete structures. Analysis and structural design (in Polish).

Rochette P., Labossière P. 2000. Axial testing of rectangular column models confined with composites, *Journal of Composites for Construction*, Vol. 4, No. 3, 129-136.

Trapko T. 2004. Load capacity of RC columns strengthening with CFRP strips and wraps. PhD. dissertation, Rapport series PRE 22/04, Institute of Building Engineering, Wroclaw University of Technology, 177 pp (in Polish).

Woliński Sz. 2000. Structural dimensioning of reinforced concrete elements with assumed probability of failure. *XLVI Konferencja naukowa Komitetu Inżynierii Lądowej i Wodnej PAN i Komitetu Nauki PZITB*, Krynica, September, p. 159-166 (in Polish).